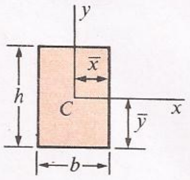
Tablas del Libro: "*Mecanica de los Materiales de los Materiales*" de Gere- Timoshenko

APÉNDICE D

Propiedades de áreas planas

Notación: A = área
$\bar{x}$, $\bar{y}$ = distancias al centroide C
I_x, I_y = momentos de inercia con respecto a los ejes x y y, respectivamente
I_{xy} = producto de inercia con respecto a los ejes x y y
$I_p = I_x + I_y$ = momento polar de inercia
I_{BB} = momento de inercia con respecto al eje B-B

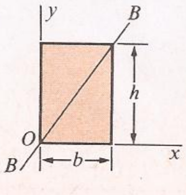
1 

Rectángulo (Origen de los ejes en el centroide.)

$$A = bh \qquad \bar{x} = \frac{b}{2} \qquad \bar{y} = \frac{h}{2}$$

$$I_x = \frac{bh^3}{12} \qquad I_y = \frac{hb^3}{12}$$

$$I_{xy} = 0 \qquad I_p = \frac{bh}{12}(h^2 + b^2)$$

2 

Rectángulo (Origen de los ejes en una esquina.)

$$I_x = \frac{bh^3}{3} \qquad I_y = \frac{hb^3}{3}$$

$$I_{xy} = \frac{b^2h^2}{4} \qquad I_p = \frac{bh}{3}(h^2 + b^2) \qquad I_{BB} = \frac{b^3h^3}{6(b^2 + h^2)}$$

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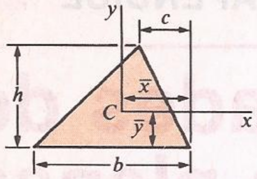

Triángulo (Origen de los ejes en el centroide.)

$$A = \frac{bh}{2} \qquad \bar{x} = \frac{b+c}{3} \qquad \bar{y} = \frac{h}{3}$$

$$I_x = \frac{bh^3}{36} \qquad I_y = \frac{bh}{36}(b^2 - bc + c^2)$$

$$I_{xy} = \frac{bh^2}{72}(b - 2c) \qquad I_p = \frac{bh}{36}(h^2 + b^2 - bc + c^2)$$

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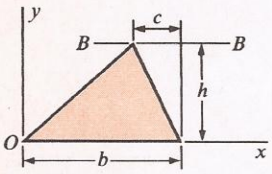

Triángulo (Origen de los ejes en el vértice.)

$$I_x = \frac{bh^3}{12} \qquad I_y = \frac{bh}{12}(3b^2 - 3bc + c^2)$$

$$I_{xy} = \frac{bh^2}{24}(3b - 2c) \qquad I_{BB} = \frac{bh^3}{4}$$

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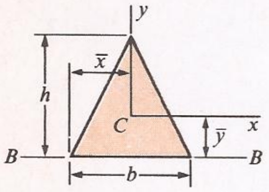

Triángulo isósceles (Origen de los ejes en el centroide.)

$$A = \frac{bh}{2} \qquad \bar{x} = \frac{b}{2} \qquad \bar{y} = \frac{h}{3}$$

$$I_x = \frac{bh^3}{36} \qquad I_y = \frac{hb^3}{48} \qquad I_{xy} = 0$$

$$I_p = \frac{bh}{144}(4h^2 + 3b^2) \qquad I_{BB} = \frac{bh^3}{12}$$

(Nota: Para un triángulo equilátero, $h = \sqrt{3}b/2$.)

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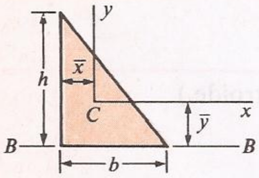

Triángulo rectángulo (Origen de los ejes en el centroide.)

$$A = \frac{bh}{2} \qquad \bar{x} = \frac{b}{3} \qquad \bar{y} = \frac{h}{3}$$

$$I_x = \frac{bh^3}{36} \qquad I_y = \frac{hb^3}{36} \qquad I_{xy} = -\frac{b^2h^2}{72}$$

$$I_p = \frac{bh}{36}(h^2 + b^2) \qquad I_{BB} = \frac{bh^3}{12}$$

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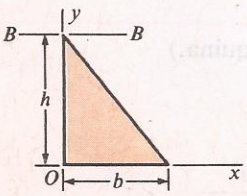

Triángulo rectángulo (Origen de los ejes en el vértice.)

$$I_x = \frac{bh^3}{12} \qquad I_y = \frac{hb^3}{12} \qquad I_{xy} = \frac{b^2h^2}{24}$$

$$I_p = \frac{bh}{12}(h^2 + b^2) \qquad I_{BB} = \frac{bh^3}{4}$$

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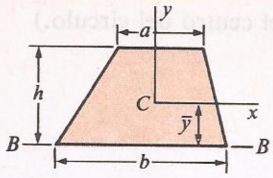

Trapezoide (Origen de los ejes en el centroide.)

$$A = \frac{h(a + b)}{2} \qquad \bar{y} = \frac{h(2a + b)}{3(a + b)}$$

$$I_x = \frac{h^3(a^2 + 4ab + b^2)}{36(a + b)} \qquad I_{BB} = \frac{h^3(3a + b)}{12}$$

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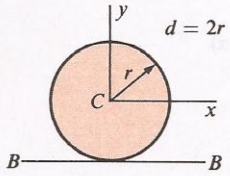

Círculo (Origen de los ejes en el centro.)

$$A = \pi r^2 = \frac{\pi d^2}{4} \qquad I_x = I_y = \frac{\pi r^4}{4} = \frac{\pi d^4}{64}$$

$$I_{xy} = 0 \qquad I_p = \frac{\pi r^4}{2} = \frac{\pi d^4}{32} \qquad I_{BB} = \frac{5\pi r^4}{4} = \frac{5\pi d^4}{64}$$

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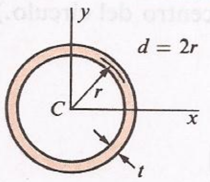

Anillo circular (Origen de los ejes en el centro.)
Fórmulas aproximadas para el caso cuando t es pequeño.

$$A = 2\pi r t = \pi d t \qquad I_x = I_y = \pi r^3 t = \frac{\pi d^3 t}{8}$$

$$I_{xy} = 0 \qquad I_p = 2\pi r^3 t = \frac{\pi d^3 t}{4}$$

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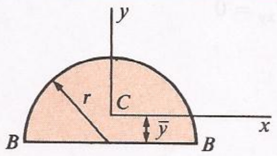

Semicírculo (Origen de los ejes en el centroide.)

$$A = \frac{\pi r^2}{2} \qquad \bar{y} = \frac{4r}{3\pi}$$

$$I_x = \frac{(9\pi^2 - 64)r^4}{72\pi} \approx 0.1098r^4 \qquad I_y = \frac{\pi r^4}{8}$$

$$I_{xy} = 0 \qquad I_{BB} = \frac{\pi r^4}{8}$$

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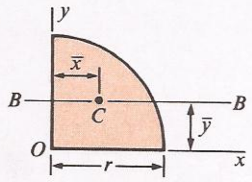


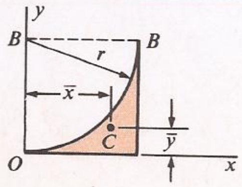
Cuadrante de círculo (Origen de los ejes en el centro del círculo.)

$$A = \frac{\pi r^2}{4} \qquad \bar{x} = \bar{y} = \frac{4r}{3\pi}$$

$$I_x = I_y = \frac{\pi r^4}{16} \qquad I_{xy} = \frac{r^4}{8}$$

$$I_{BB} = \frac{(9\pi^2 - 64)r^4}{144\pi} \approx 0.05488r^4$$

13



Arco de cuadrante de círculo (Origen de los ejes en el vértice.)

$$A = \left(1 - \frac{\pi}{4}\right)r^2$$

$$\bar{x} = \frac{2r}{3(4 - \pi)} \approx 0.7766r \qquad \bar{y} = \frac{(10 - 3\pi)r}{3(4 - \pi)} \approx 0.2234r$$

$$I_x = \left(1 - \frac{5\pi}{16}\right)r^4 \approx 0.01825r^4 \qquad I_y = I_{BB} = \left(\frac{1}{3} - \frac{\pi}{16}\right)r^4 \approx 0.1370r^4$$

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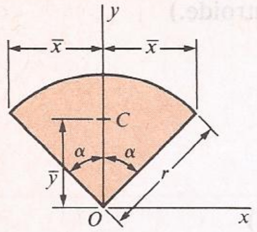
14 **Sector circular** (Origen de los ejes en el centro del círculo.)



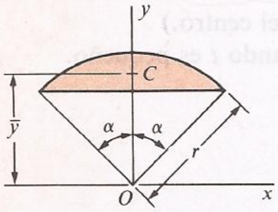
α = ángulo en radianes $\left(\alpha \leq \dfrac{\pi}{2}\right)$

$$A = \alpha r^2 \qquad \bar{x} = r \operatorname{sen} \alpha \qquad \bar{y} = \frac{2r \operatorname{sen} \alpha}{3\alpha}$$

$$I_x = \frac{r^4}{4}(\alpha + \operatorname{sen} \alpha \cos \alpha) \qquad I_y = \frac{r^4}{4}(\alpha - \operatorname{sen} \alpha \cos \alpha)$$

$$I_{xy} = 0 \qquad I_p = \frac{\alpha r^4}{2}$$

15 **Segmento circular** (Origen de los ejes en el centro del círculo.)



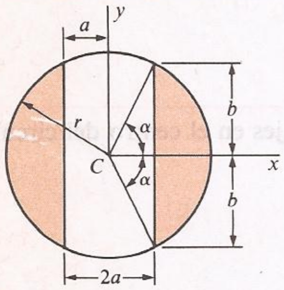
α = ángulo en radianes $\left(\alpha \leq \dfrac{\pi}{2}\right)$

$$A = r^2(\alpha - \operatorname{sen} \alpha \cos \alpha) \qquad \bar{y} = \frac{2r}{3}\left(\frac{\operatorname{sen}^3 \alpha}{\alpha - \operatorname{sen} \alpha \cos \alpha}\right)$$

$$I_x = \frac{r^4}{4}(\alpha - \operatorname{sen} \alpha \cos \alpha + 2 \operatorname{sen}^3 \alpha \cos \alpha) \qquad I_{xy} = 0$$

$$I_y = \frac{r^4}{12}(3\alpha - 3 \operatorname{sen} \alpha \cos \alpha - 2 \operatorname{sen}^3 \alpha \cos \alpha)$$

16 **Círculo con núcleo removido** (Origen de los ejes en el centro del círculo.)



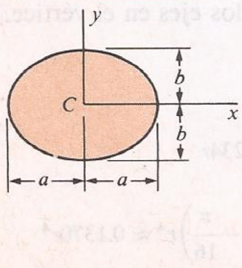
α = ángulo en radianes $\left(\alpha \leq \dfrac{\pi}{2}\right)$

$$\alpha = \arccos \frac{a}{r} \qquad b = \sqrt{r^2 - a^2}$$

$$A = 2r^2\left(\alpha - \frac{ab}{r^2}\right) \qquad I_{xy} = 0$$

$$I_x = \frac{r^4}{6}\left(3\alpha - \frac{3ab}{r^2} - \frac{2ab^3}{r^4}\right) \qquad I_y = \frac{r^4}{2}\left(\alpha - \frac{ab}{r^2} + \frac{2ab^3}{r^4}\right)$$

17 **Elipse** (Origen de los ejes en el centroide.)



$$A = \pi ab \qquad I_x = \frac{\pi ab^3}{4} \qquad I_y = \frac{\pi ba^3}{4}$$

$$I_{xy} = 0 \qquad I_p = \frac{\pi ab}{4}(b^2 + a^2)$$

$$\text{Perímetro} \approx \pi[1.5(a + b) - \sqrt{ab}]$$

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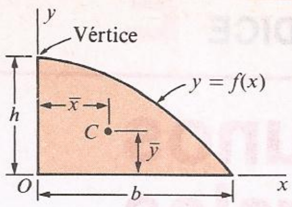
18 **Semisegmento parabólico** (Origen de los ejes en la esquina.)

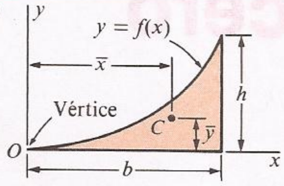


$$y = f(x) = h\left(1 - \frac{x^2}{b^2}\right)$$

$$A = \frac{2bh}{3} \qquad \bar{x} = \frac{3b}{8} \qquad \bar{y} = \frac{2h}{5}$$

$$I_x = \frac{16bh^3}{105} \qquad I_y = \frac{2hb^3}{15} \qquad I_{xy} = \frac{b^2h^2}{12}$$

19 **Arco parabólico** (Origen de los ejes en el vértice.)

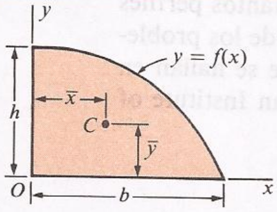


$$y = f(x) = \frac{hx^2}{b^2}$$

$$A = \frac{bh}{3} \qquad \bar{x} = \frac{3b}{4} \qquad \bar{y} = \frac{3h}{10}$$

$$I_x = \frac{bh^3}{21} \qquad I_y = \frac{hb^3}{5} \qquad I_{xy} = \frac{b^2h^2}{12}$$

20 **Semisegmento de grado n-ésimo** (Origen de los ejes en la esquina.)



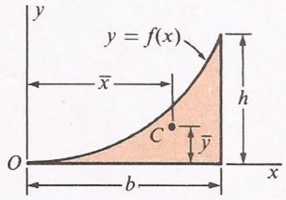
$$y = f(x) = h\left(1 - \frac{x^n}{b^n}\right) \qquad n > 0$$

$$A = bh\left(\frac{n}{n+1}\right) \qquad \bar{x} = \frac{b(n+1)}{2(n+2)} \qquad \bar{y} = \frac{hn}{2n+1}$$

$$I_x = \frac{2bh^3n^3}{(n+1)(2n+1)(3n+1)}$$

$$I_y = \frac{hb^3n}{3(n+3)} \qquad I_{xy} = \frac{b^2h^2n^2}{4(n+1)(n+2)}$$

21 **Arco de grado n-ésimo** (Origen de los ejes en el vértice.)

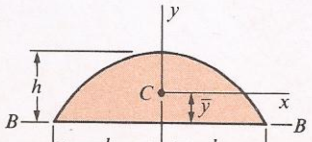


$$y = f(x) = \frac{hx^n}{b^n} \qquad n > 0$$

$$A = \frac{bh}{n+1} \qquad \bar{x} = \frac{b(n+1)}{n+2} \qquad \bar{y} = \frac{h(n+1)}{2(2n+1)}$$

$$I_x = \frac{bh^3}{3(3n+1)} \qquad I_y = \frac{hb^3}{n+3} \qquad I_{xy} = \frac{b^2h^2}{4(n+1)}$$

22 **Onda senoidal** (Origen de los ejes en el centroide.)



$$A = \frac{4bh}{\pi} \qquad \bar{y} = \frac{\pi h}{8}$$

$$I_x = \left(\frac{8}{9\pi} - \frac{\pi}{16}\right)bh^3 \approx 0.08659bh^3 \qquad I_y = \left(\frac{4}{\pi} - \frac{32}{\pi^3}\right)hb^3 \approx 0.2412hb^3$$

$$I_{xy} = 0 \qquad I_{BB} = \frac{8bh^3}{9\pi}$$

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