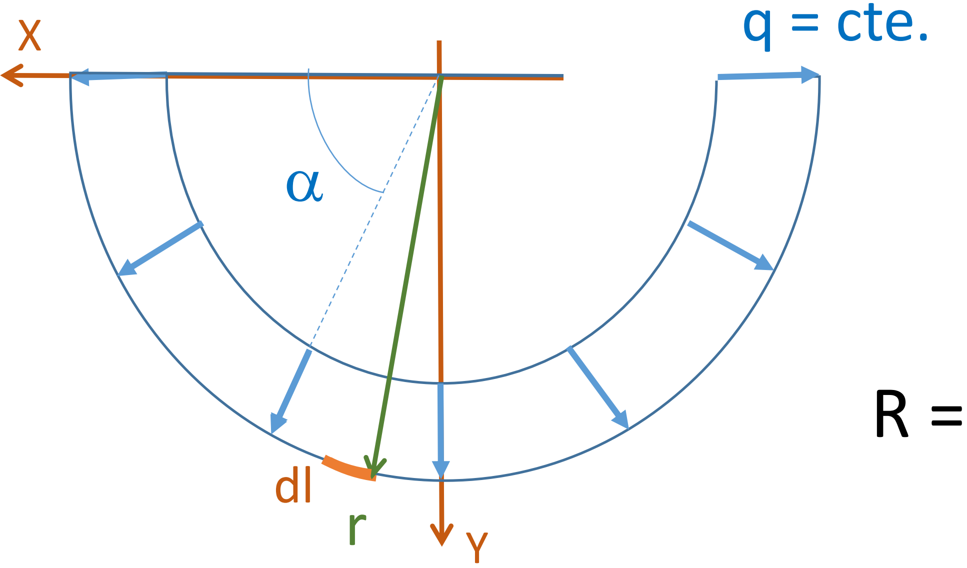
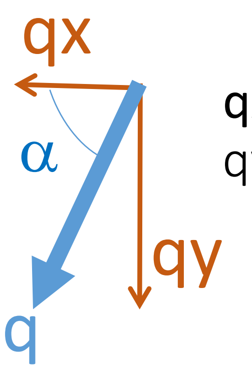
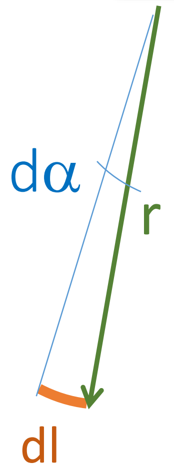
Fuerzas Distribuidas – Corona Circular

Caso 1: $\alpha = 180° = 2\pi$ rad





qx = q . Cosα
qy = q . Senα



dl = r .dα

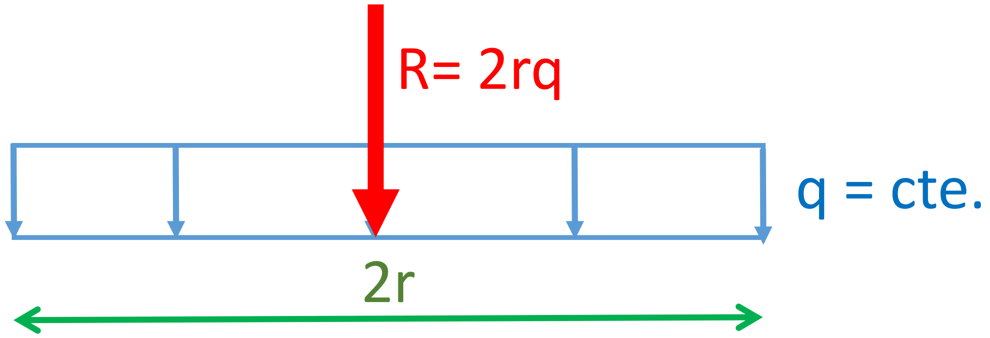
$$R = \int_0^{\pi} q\, dl = \int_0^{\pi} q\, r\, d\alpha$$

$$Rx = \int_0^{\pi} q_x\, r d\alpha = \int_0^{\pi} q\, cosa\, r\, d\alpha = r\, q \int_0^{\pi} \cos\alpha\, d\alpha = r\, q\, [sen\alpha\,]\,_0^{\pi} = r\, q\, [\,0 - 0\,] = 0$$

$$Ry = \int_0^{\pi} q_y\, r d\alpha = \int_0^{\pi} q\, sen\alpha\, r\, d\alpha = rq \int_0^{\pi} sen\alpha\, d\alpha = r\, q[-cosa\,]\,_0^{\pi} = r\, q[-(\,-1\ - 1)\,] = 2rq$$

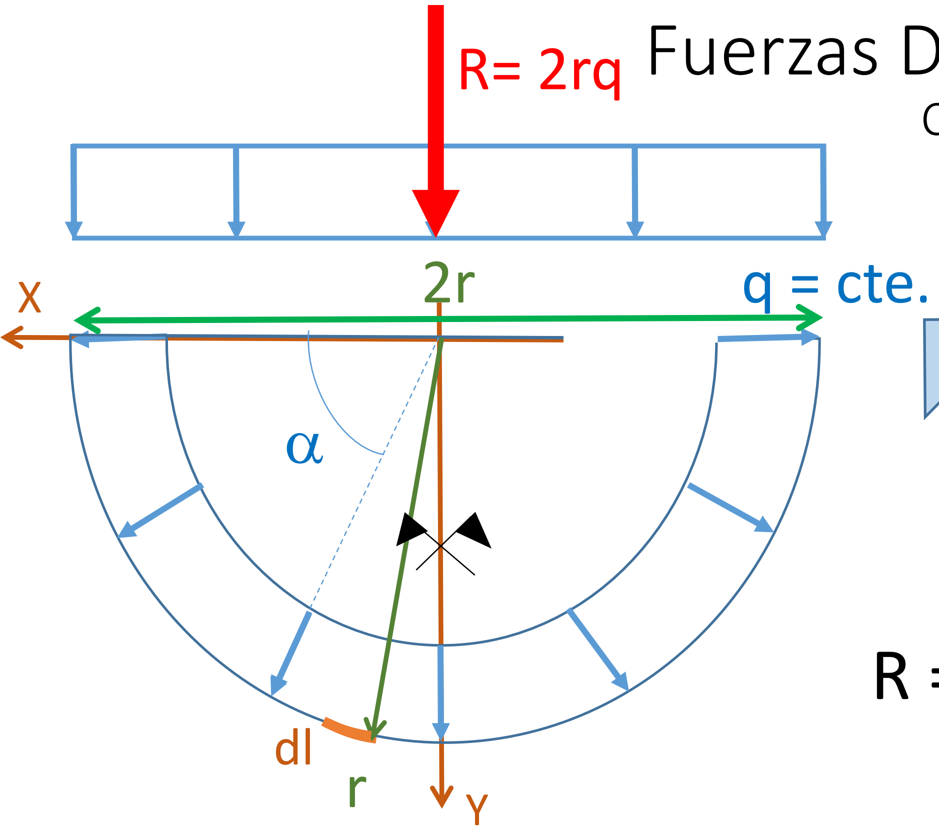
$$R = \sqrt{Rx^2 + Ry^2} = Ry = 2rq$$

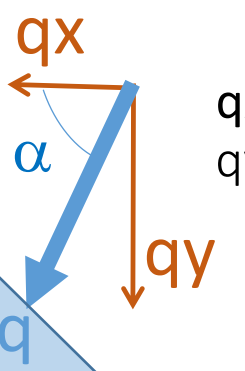
Equivalente a una carga q=cte. En una longitud de "2r" =>



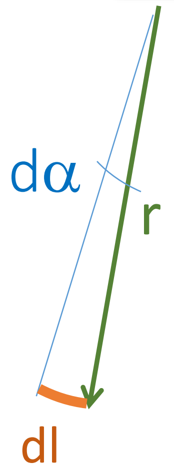
Fuerzas Distribuidas – Corona Circular

Caso 1: $\alpha = 180° = 2\pi$ rad





qx = q . Cosα
qy = q . Senα



dl = r .dα

$$R = \int_0^{\pi} q\, dl = \int_0^{\pi} q\, r\, d\alpha$$

$$Rx = \int_0^{\pi} q_x\, r d\alpha = \int_0^{\pi} q \cos\alpha\, r\, d\alpha = r\, q \int_0^{\pi} \cos\alpha\, d\alpha = r\, q\, [\operatorname{sen}\alpha\,]_0^{\pi} = r\, q\, [\,0 - 0\,] = 0$$

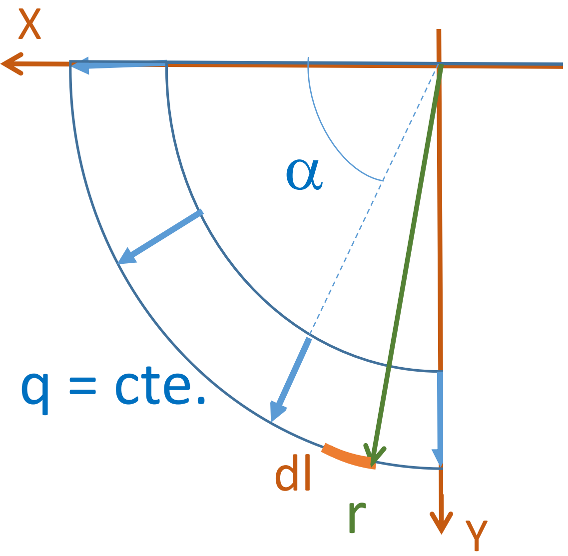
$$Ry = \int_0^{\pi} q_y\, r d\alpha = \int_0^{\pi} q\, \operatorname{sen}\alpha\, r\, d\alpha = rq \int_0^{\pi} \operatorname{sen}\alpha\, d\alpha = r\, q[-\cos\alpha\,]_0^{\pi} = r\, q[-(\,-1\; - 1)\,] = 2rq$$

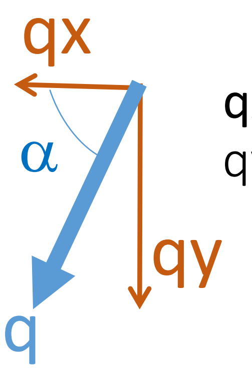
$$R = \sqrt{Rx^2 + Ry^2} = Ry = 2rq$$

Equivalente a una carga q=cte. En una longitud de "2r" =>

Fuerzas Distribuidas – Corona Circular

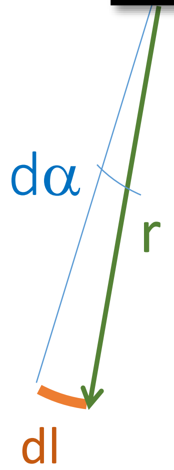
Caso 2: α = 90° = π rad





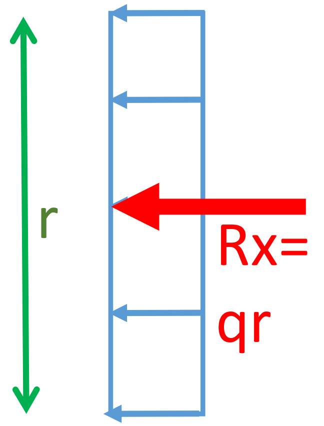
$q_x = q \cdot \cos\alpha$

$q_y = q \cdot \operatorname{sen}\alpha$



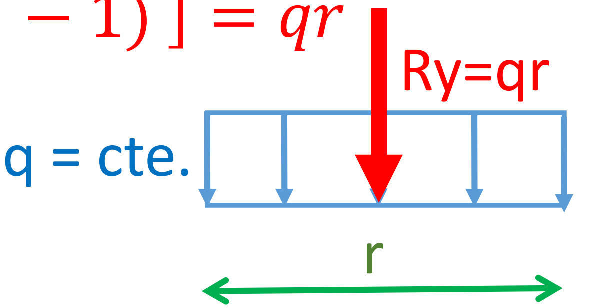
$dl = r \cdot d\alpha$

$$R = \int_0^{\pi/2} q\, dl = \int_0^{\pi/2} q\, r\, d\alpha$$



$$Rx = \int_0^{\pi/2} q_x\, r d\alpha = \int_0^{\pi/2} q\, cos\alpha\, r\, d\alpha = q\, r\, [sen\alpha\,]_0^{\pi/2} = q\, r\, [\,1\, - 0\,] = qr$$

$$Ry = \int_0^{\pi/2} q_y\, r d\alpha = \int_0^{\pi/2} q\, sen\alpha\, r\, d\alpha = q\, r\, [-cos\alpha\,]_0^{\pi/2} = q\, r\, [-(\,0\, - 1)\,] = qr$$



$$R = \sqrt{Rx^2 + Ry^2} = \sqrt{2}\, rq$$

Fuerzas Distribuidas – Corona Circular

Caso 2: α = 90° = π rad

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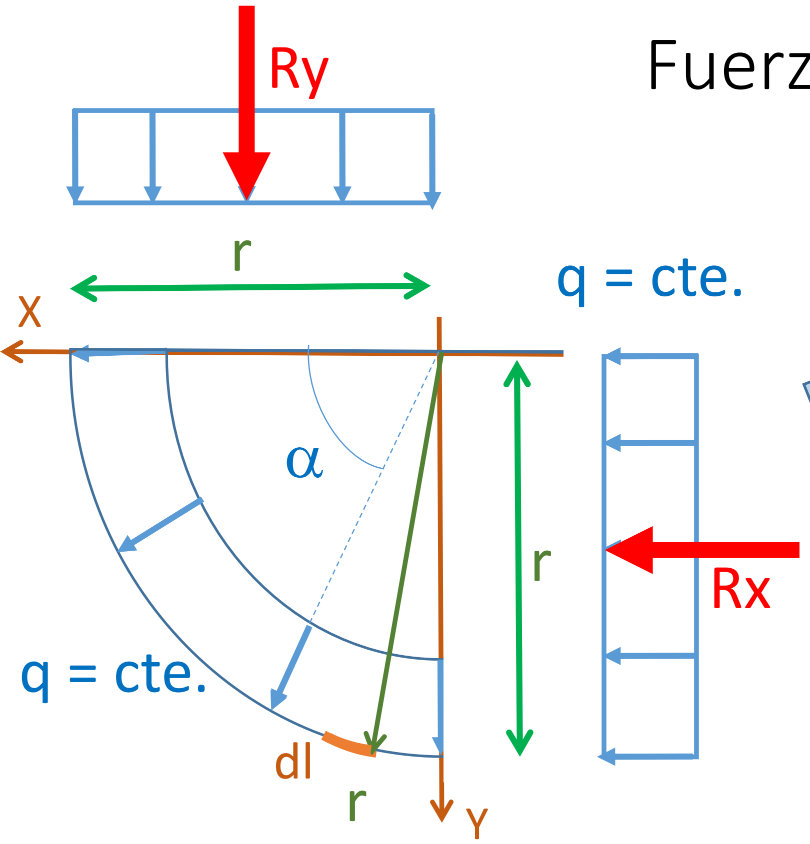
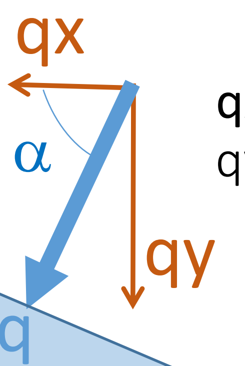
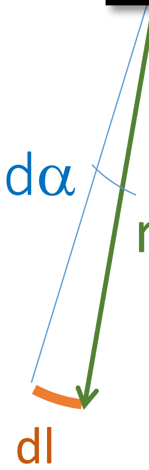
Ejercicios Tema N° 2: Fuerzas Distribuidas





$qx = q \cdot Cos\alpha$

$qy = q \cdot Sen\alpha$



$dl = r \cdot d\alpha$

$$R = \int_0^{\pi/2} q\, dl = \int_0^{\pi/2} q\, r\, d\alpha$$

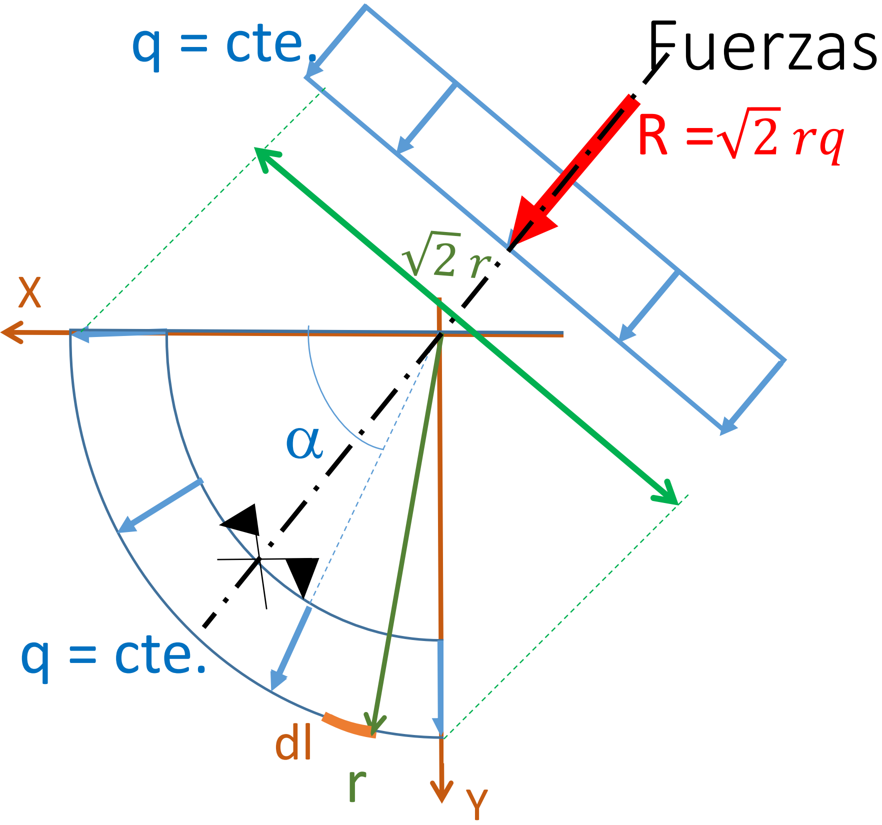
$$Rx = \int_0^{\pi/2} q_x\, r d\alpha = \int_0^{\pi/2} q\, cos\alpha\, r\, d\alpha = q\, r\, [sen\alpha\,]_0^{\pi/2} = q\, r\, [\,1\ - 0\,] = qr$$

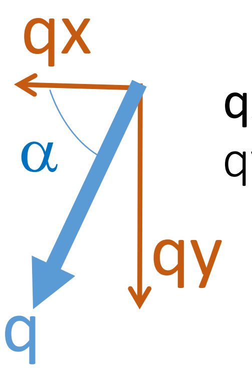
$$Ry = \int_0^{\pi/2} q_y\, r d\alpha = \int_0^{\pi/2} q\, sen\alpha\, r\, d\alpha = q\, r\, [-cos\alpha\,]_0^{\pi/2} = q\, r\, [-(\,0\ - 1)\,] = qr$$

$$R = \sqrt{Rx^2 + Ry^2} = \sqrt{2}\, rq$$

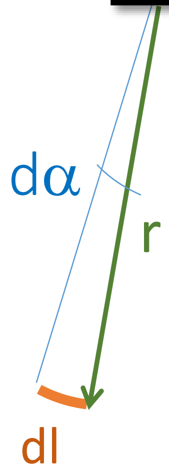
Fuerzas Distribuidas – Corona Circular

Caso 2: α = 90° = π rad





qx = q . Cosα
qy = q . Senα



dl = r .dα

$$R = \int_0^{\pi/2} q\, dl = \int_0^{\pi/2} q\, r\, d\alpha$$

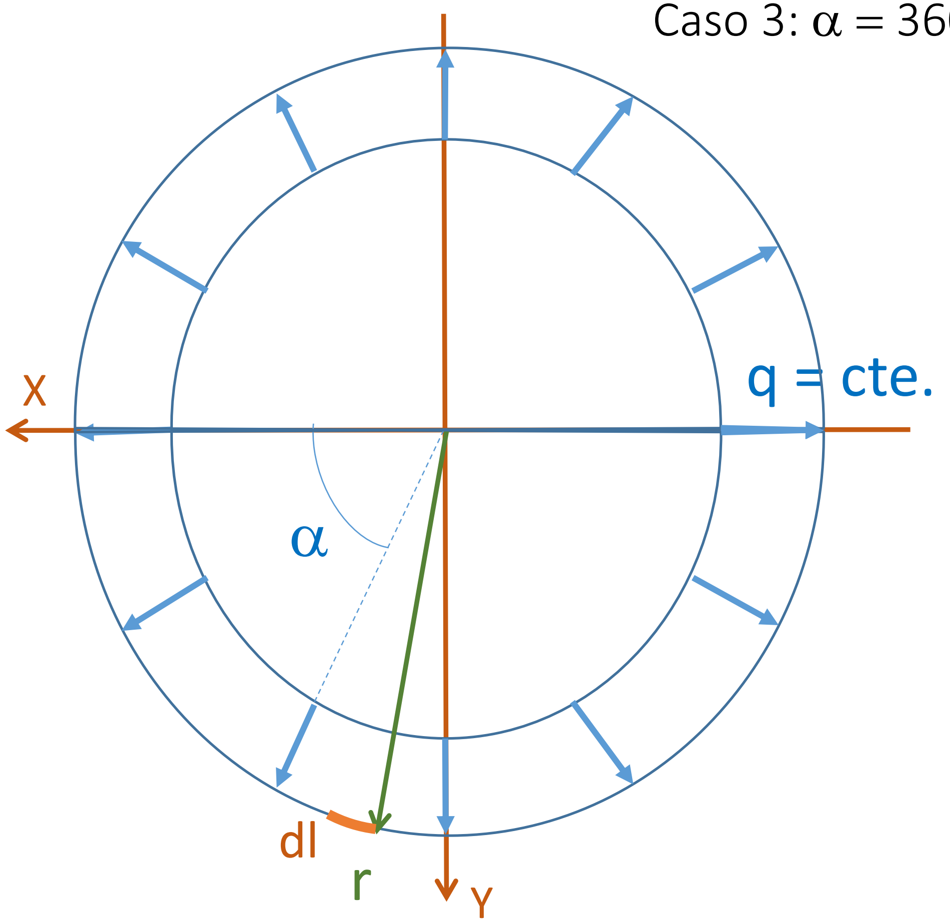
$$Rx = \int_0^{\pi/2} q_x\, r d\alpha = \int_0^{\pi/2} q\, cos\alpha\, r\, d\alpha = q\, r\, [sen\alpha\,]\,_0^{\pi/2} = q\, r\, [\,1\, - 0\,] = qr$$

$$Ry = \int_0^{\pi/2} q_y\, r d\alpha = \int_0^{\pi/2} q\, sen\alpha\, r\, d\alpha = q\, r\, [-cos\alpha\,]\,_0^{\pi/2} = q\, r\, [-(\,0\, - 1)\,] = qr$$

$$R = \sqrt{Rx^2 + Ry^2} = \sqrt{2}\, rq$$

Fuerzas Distribuidas – Corona Circular

Caso 3: α = 360° = 2π rad



$$R = \int_0^{2\pi} q\, dl = \int_0^{2\pi} q\, r\, d\alpha$$

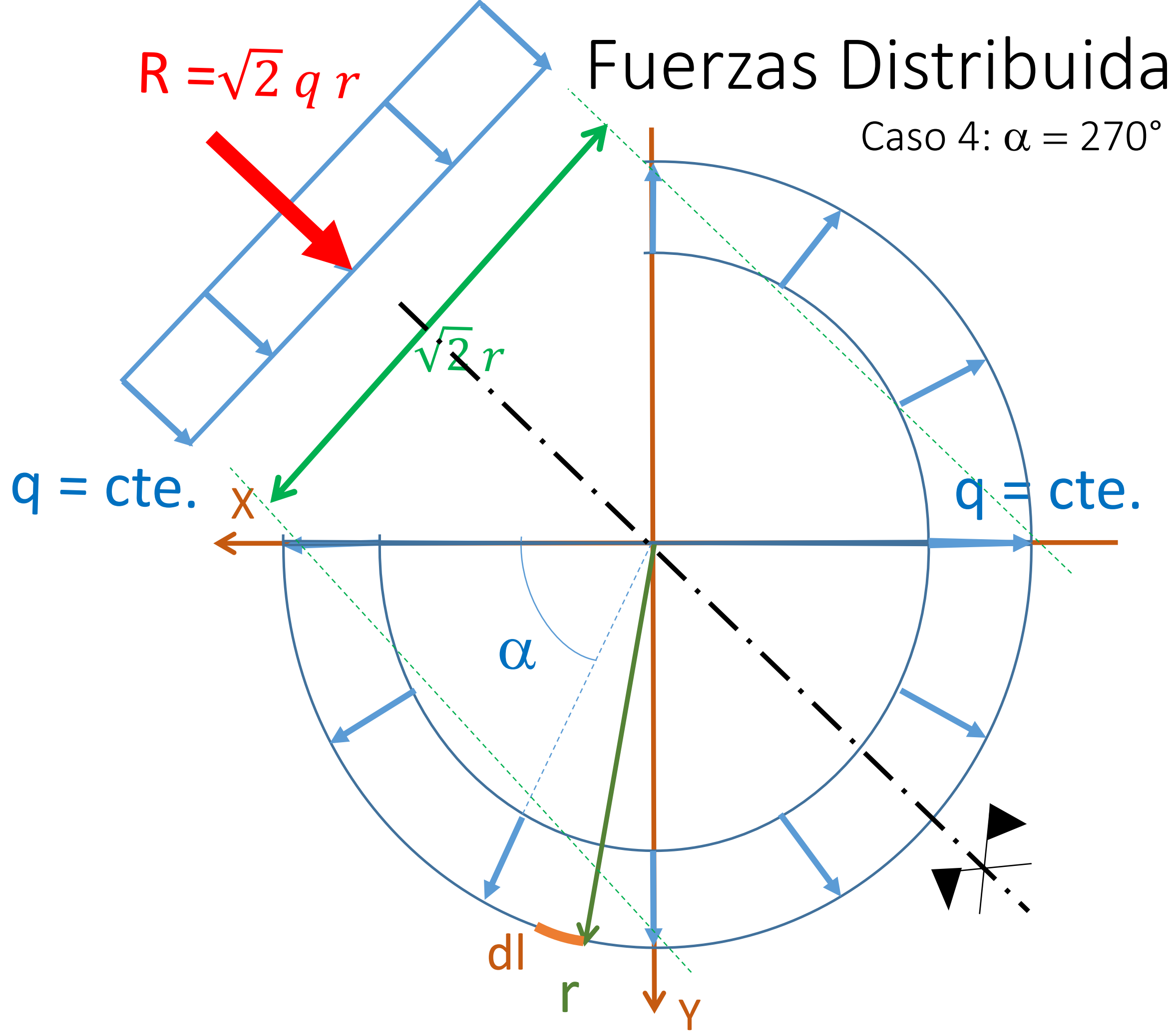
$$Rx = \int_0^{2\pi} q_x\, r d\alpha = \int_0^{2\pi} q\, cos\alpha\, r\, d\alpha = q\, r\, [sen\alpha\,]\,_0^{2\pi} = q\, r\, [\, 0\ - 0\,] = 0$$

$$Ry = \int_0^{2\pi} q_y\, r d\alpha = \int_0^{2\pi} q\, sen\alpha\, r\, d\alpha = q\, r\, [-cos\alpha\,]\,_0^{2\pi} = q\, r\, [-(\,1\ - 1)\,] = 0$$

$$R = \sqrt{Rx^2 + Ry^2} = 0$$

Fuerzas Distribuidas – Corona Circular

Caso 4: $\alpha = 270° = 3/2\ \pi$ rad



$$R = \int_0^{3/2\pi} q\,dl = \int_0^{3/2\pi} q\,r\,d\alpha$$

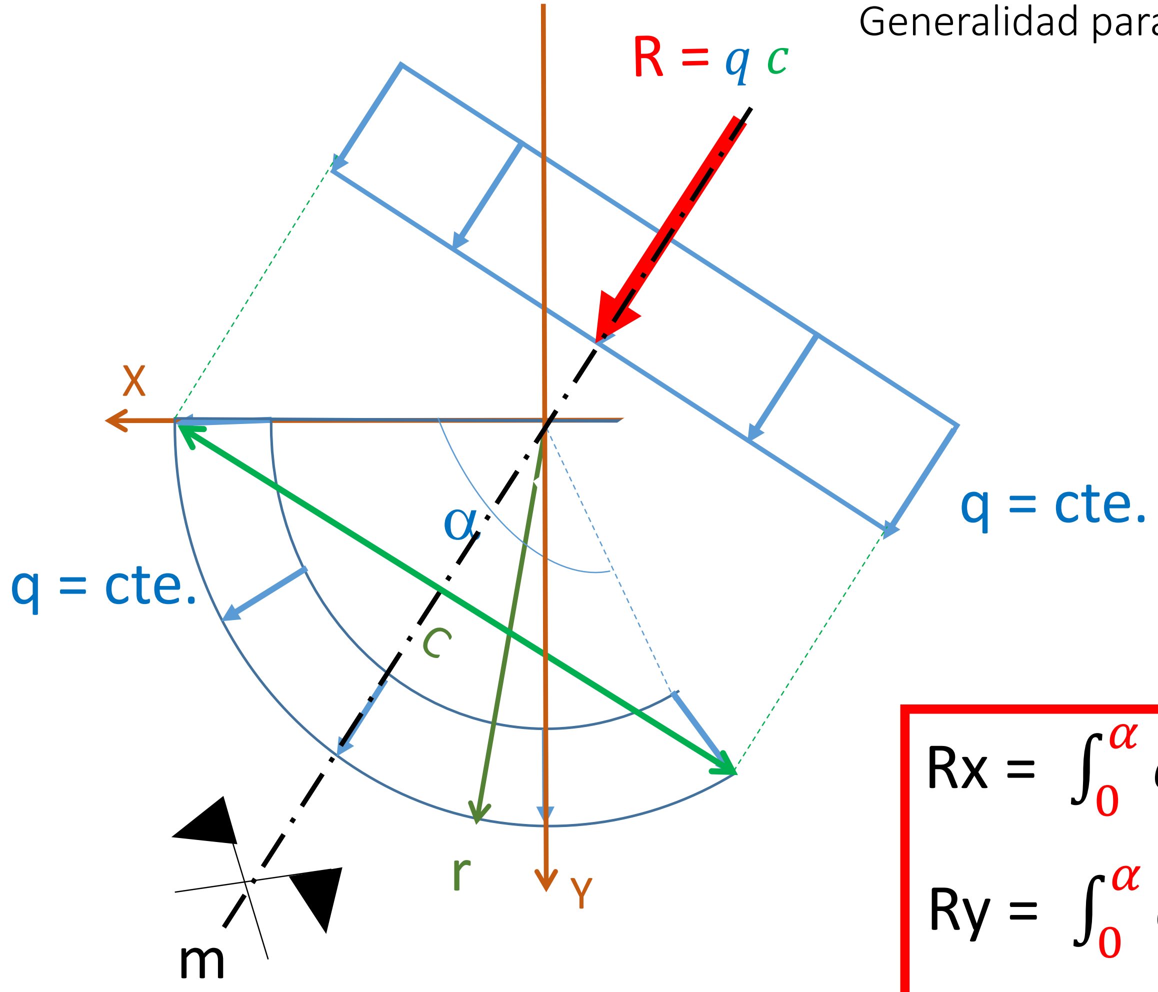
$$Rx = \int_0^{3/2\pi} q_x\,r d\alpha = \int_0^{3/2\pi} q\,cos\alpha\,r\,d\alpha = q\,r\,[sen\alpha\,]\,_0^{3/2\pi} = q\,r\,[-1\ -0\,] = -q\,r$$

$$Ry = \int_0^{3/2\pi} q_y\,r d\alpha = \int_0^{3/2\pi} q\,sen\alpha\,r\,d\alpha = q\,r\,[-cos\alpha\,]\,_0^{3/2\pi} = q\,r\,[-(\,0\ -1)\,] = q\,r$$

$$R = \sqrt{Rx^2 + Ry^2} = \sqrt{2}\ q\ r$$

Fuerzas Distribuidas – Corona Circular

Generalidad para cualquier α



$$R = \int_0^{\alpha} q\, dl = \int_0^{\alpha} q\, r\, d\alpha$$

$$Rx = \int_0^{\alpha} q_x\, r d\alpha = \int_0^{\alpha} q\, cos\alpha\, r\, d\alpha = q\, r\, [sen\alpha\,]_0^{\alpha}$$

$$Ry = \int_0^{\alpha} q_y\, r d\alpha = \int_0^{\alpha} q\, sen\alpha\, r\, d\alpha = q\, r\, [-cos\alpha\,]_0^{\alpha}$$

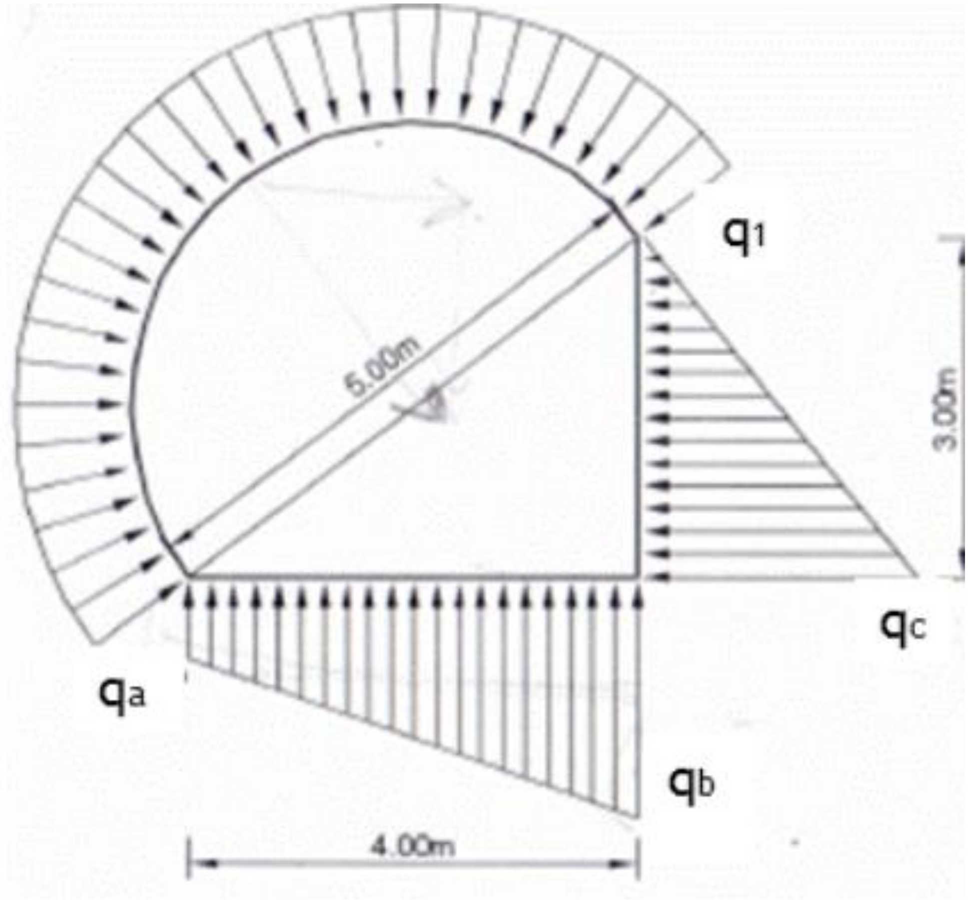
$$R = \sqrt{Rx^2 + Ry^2} = q\ c$$

m: eje de simetría de la corona/semicorona circular

C: longitud de la cuerda de la corona/semicorona circular ortogonal al eje de simetría "m"

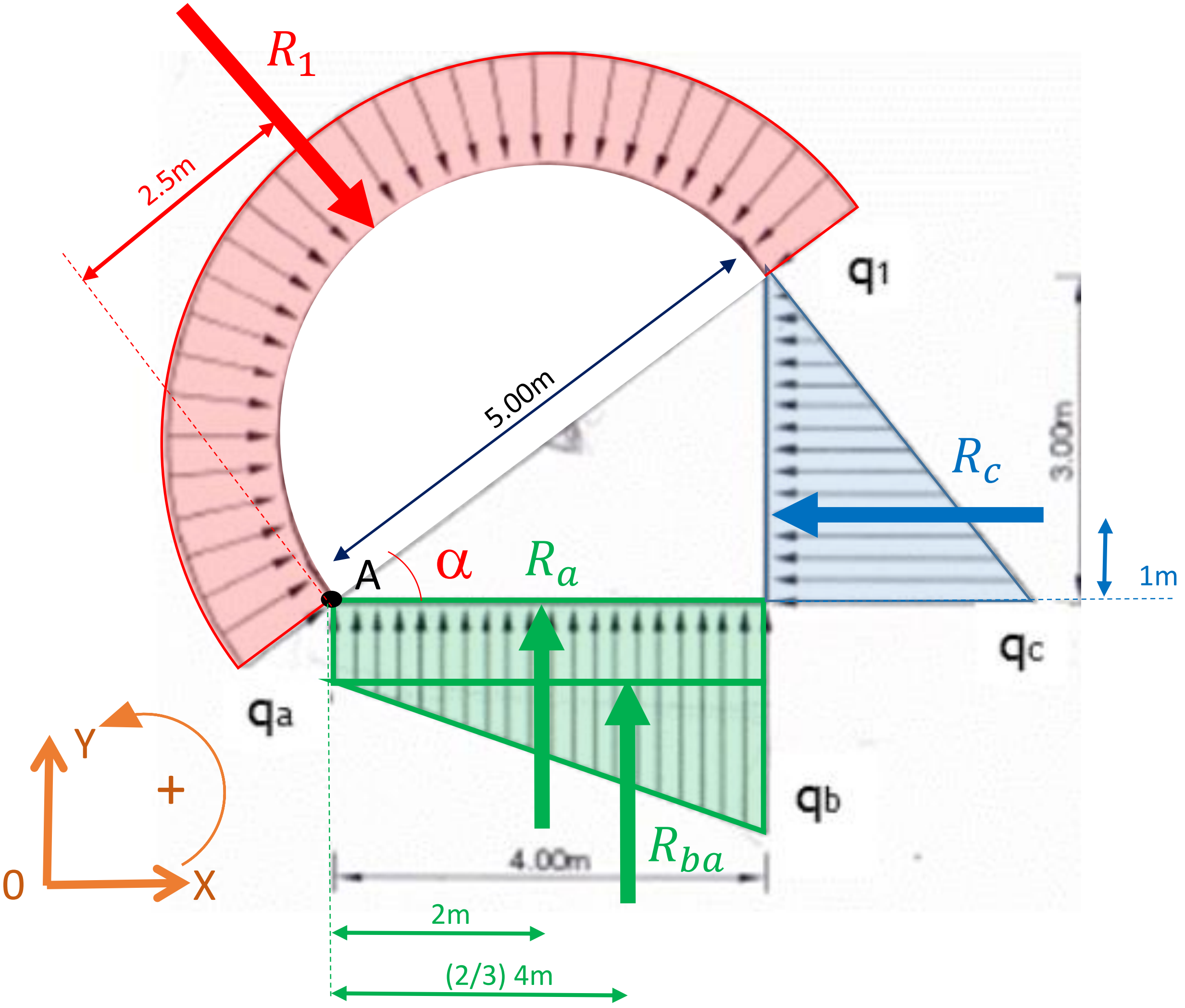
Fuerzas Distribuidas – Corona Circular

Ejercicio 10: Hallar los valores de qa, qb y qc para que la chapa de la figura se encuentreen equilibrio. Datos: q1 = 15KN/m

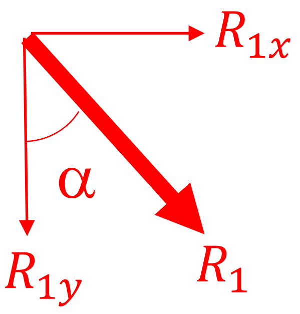


Fuerzas Distribuidas – Corona Circular

Ejercicio 10: Hallar los valores de qa, qb y qc para que la chapa de la figura se encuentre en equilibrio. Datos: q1 = 15KN/m



a- Poner en evidencia las resultantes de las fuerzas distribuidas:



$$R_1 = q_1 \,.\, 5m = \frac{15KN}{m} . \, 5m = 75KN$$

$$R_{1x} = R_1 \,.\, sen\alpha = R_1 . \frac{3m}{5m} = 45KN$$

$$R_{1y} = R_1 \,.\, cos\alpha = R_1 . \frac{4m}{5m} = 60KN$$

$$R_a = q_a \,.\, 4m$$

$$R_{ba} = (q_b - q_a). 4m \,.\, 1/2$$

$$R_c = q_c . 3m \,.\, 1/2$$

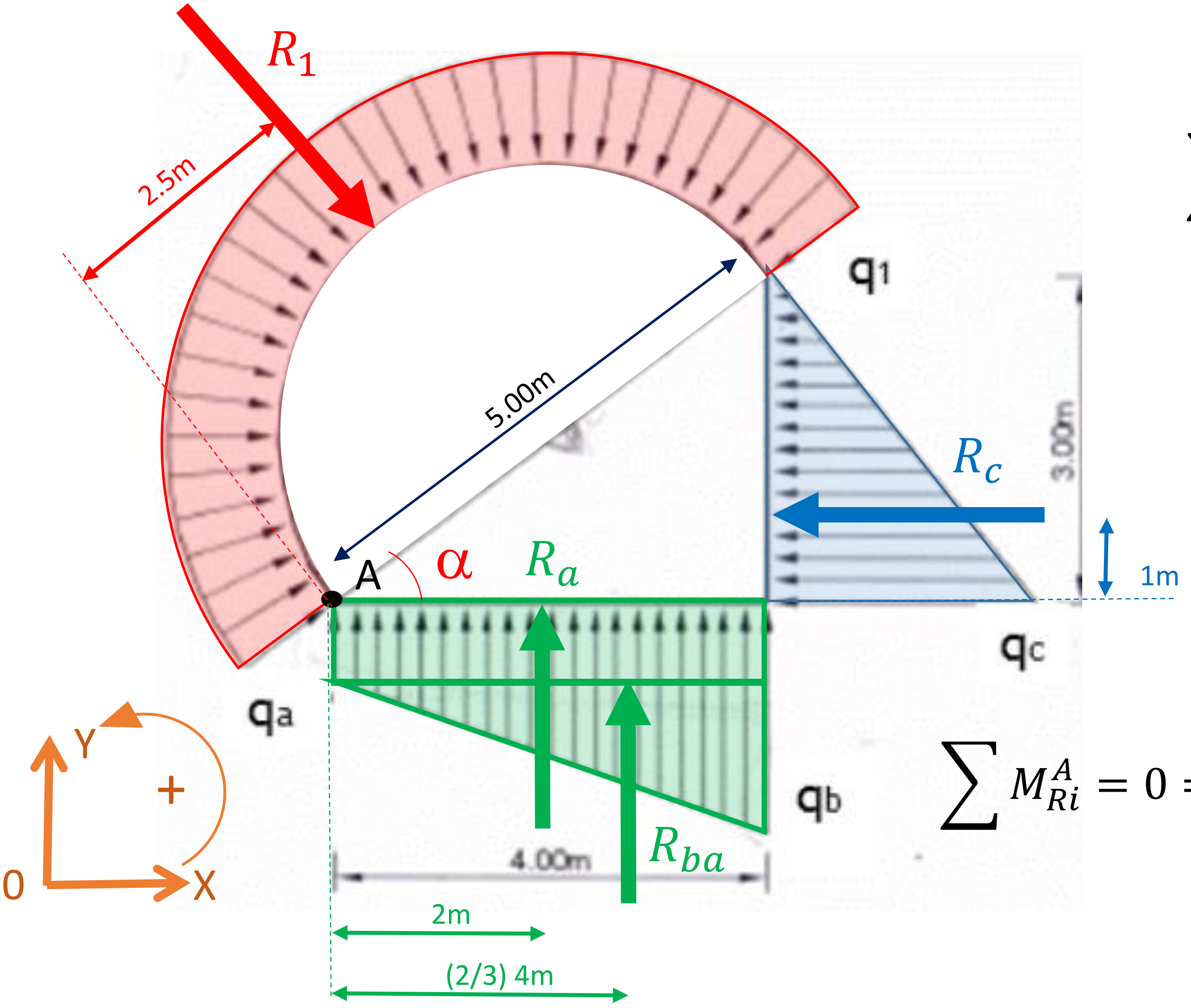
b- Para que la chapa se encuentre en equilibrio se debe cumplir:

$$\sum R_i = 0 \quad (1)$$

$$\sum MR_i^A = 0 \quad (2)$$

Fuerzas Distribuidas – Corona Circular

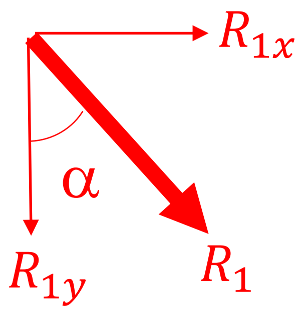
Ejercicio 10: Hallar los valores de qa, qb y qc para que la chapa de la figura se encuentre en equilibrio. Datos: q1 = 15KN/m



De (1)

$$\sum R_{ix} = 0 \Rightarrow R_{1x} - R_c = 0 \Rightarrow R_c = R_{1x} \Rightarrow$$

$$\Rightarrow q_c.\frac{3m}{2} = 45KN \Rightarrow \boxed{q_c = 30KN/m}$$



$$\sum R_{iy} = 0 \Rightarrow -R_{1y} + R_a + R_{ba} = 0 \Rightarrow$$

$$\Rightarrow R_a + R_{ba} = R_{1y} = 60KN/m \quad \text{A}$$

De (2)

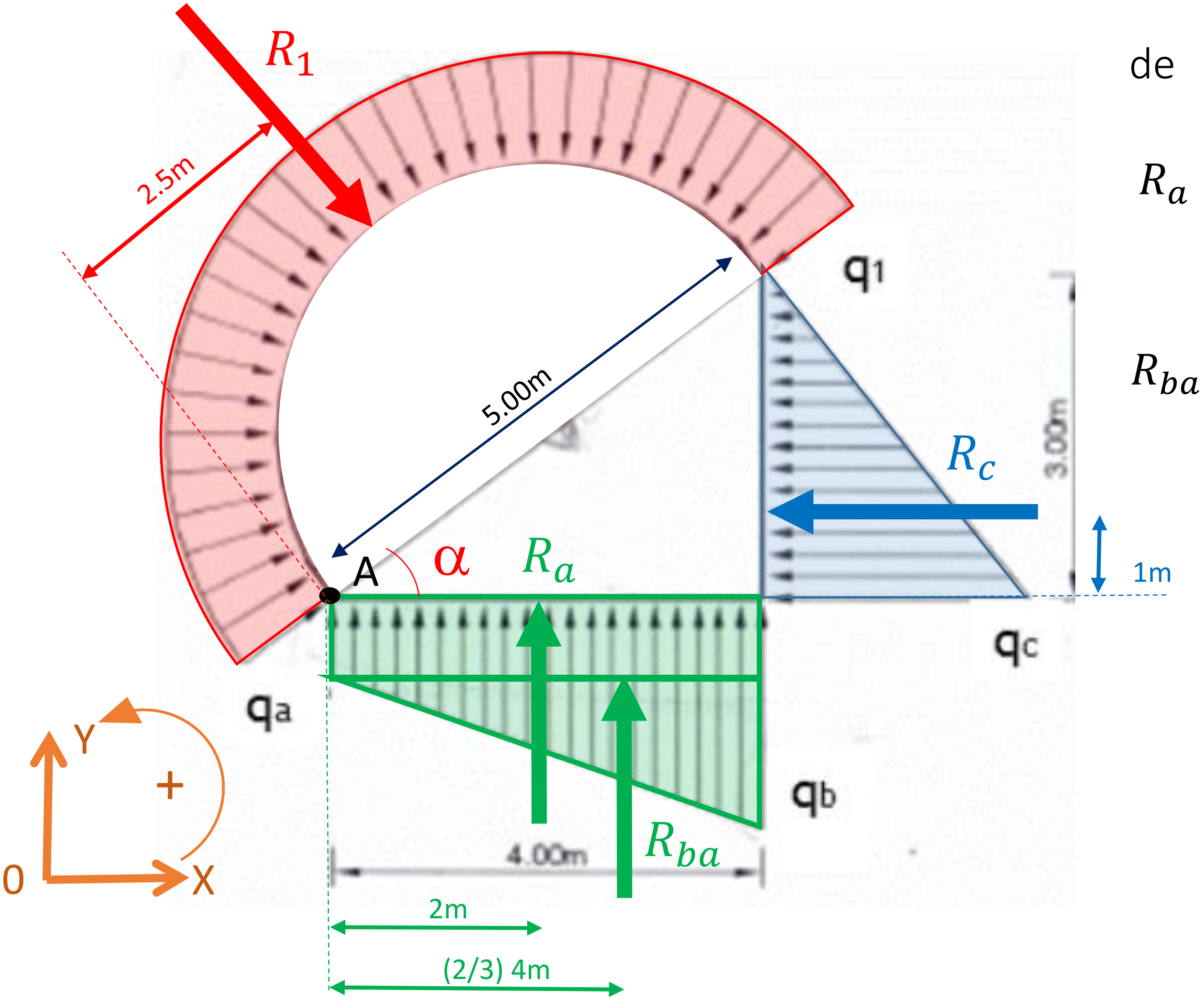
$$\sum M_{Ri}^{A} = 0 \Rightarrow -R_1\,.2.5m + R_a.2m + R_{ba}.\left(\frac{2}{3}.4m\right) + R_c.1m = 0$$

$$\Rightarrow -75KN\,.2.5m + R_a.2m + R_{ba}.\left(\frac{8}{3}\right)m + 45KN = 0$$

$$\Rightarrow R_a.2m + R_{ba}\left(\frac{8}{3}\right)m = 142.5KNm \quad \text{B}$$

Fuerzas Distribuidas – Corona Circular

Ejercicio 10: Hallar los valores de qa, qb y qc para que la chapa de la figura se encuentre en equilibrio. Datos: q1 = 15KN/m

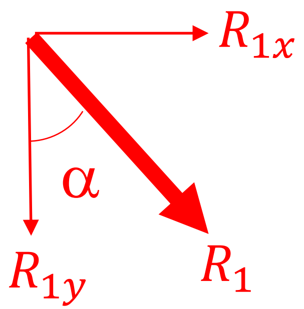


de A y B resolvemos las 2 incógnitas:

$$R_a = \frac{105}{4} KN \Rightarrow q_a = \frac{105}{4} KN \cdot \frac{1}{4} m = 6.562 KN/m$$

$$R_{ba} = \frac{135}{4} KN \Rightarrow q_b = \left(\frac{135}{4} \cdot \frac{1}{2}\right) + q_a$$

$$\Rightarrow q_b = 23.44 KN/m$$



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